# Synchronization in a Kuramoto Mean Field Game

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# Outline

# Dynamical System

# Outline

# SELF-ENTRAINMENT OF A POPULATION OF COUPLED NON-LINEAR OSCILLATORS

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Temporal organization of matter is a widespread phenomenon over a macroscopic world in far from thermodynamic equilibrium. A previous study on chemical instability[1)] implies that a simplest nontrivial model for a temporally organized system may be represented by a macroscopic self-sustained oscillator Q obeying the equation of motion

$$\dot{Q} = (i\omega + \alpha)Q - \beta|Q|^2 Q \,, \qquad \alpha,\beta > 0. \tag{1}$$

Consider a population of such oscillators $Q_1$, $Q_2$,$\cdots Q_N$ with various frequencies, and introduce interactions between every pair as follows.

$$\dot{Q}_s = (i\omega_s + \alpha)Q_s + \sum_{r\neq s} V_{rs}Q_r - \beta|Q_s|^2 Q_s \,, \qquad r,s = 1,\,2,\cdots N\,. \tag{2}$$

We found that it is possible to construct from (2) a soluble model for a community exhibiting mutual synchronization or self-entrainment above a certain threshold value of the coupling strength. Such a type of phase transition has been considered by Winfree[2)] without resorting to specialized models but only phenomenologically.

Kuramoto considered a population of $N$ coupled phase oscillators $\theta_t^k$ having natural frequencies $\omega^k$ distributed with a given density, and whose dynamics are governed by

$$\frac{\mathrm{d}}{\mathrm{d}t}\theta_t^k = \omega^k + \frac{\kappa}{N}\sum_{j=1}^{N}\sin(\theta_t^j - \theta_t^k), \qquad k = 1, \dots, N.$$

The following complex order parameter simplifies the equation :

$$r_t\, e^{i\,\psi_t} := \frac{1}{N}\sum_{j=1}^{N} e^{i\,\theta_t^j}. \quad \Rightarrow \quad r_t\, \sin(\psi_t - \theta_t^k) = \frac{1}{N}\sum_{j=1}^{N}\sin(\theta_t^j - \theta_t^k).$$

Hence, the equation has the form :

$$\frac{\mathrm{d}}{\mathrm{d}t}\theta_t^k = \omega^k + \kappa\, r_t\, \sin(\psi_t - \theta_t^k). \qquad k = 1, \dots, N.$$

Quoting : *From Kuramoto to Crawford : exploring the onset of synchronization in populations of coupled oscillators* by S. H. Strogatz (Pysica D, 2000).

$$\frac{\mathrm{d}}{\mathrm{d}t}\theta_t^k = \omega^k + \kappa\, r_t \sin(\psi_t - \theta_t^k). \qquad k = 1, \dots, N.$$

In this form, the mean-field character of the model becomes obvious. Each oscillator is interacting only through the mean-field quantities $r_t$ and $\psi_t$. Specifically, the phase $\theta_t^k$ is pulled toward the mean phase $\psi_t$, rather than toward the phase of any individual oscillator. Moreover, the effective strength of the coupling is proportional to the coherence $r_t$. This proportionality sets up a positive feedback loop between coupling and coherence : as the population becomes more coherent, $r_t$ grows and so the effective coupling $\kappa\, r_t$ increases, which tends to recruit even more oscillators into the synchronized pack.

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# Critical $\kappa$

There exists a critical threshold $\widehat{\kappa}_c$ (depending on the distribution of $\omega^i$s) such that :

- For all $\kappa < \widehat{\kappa}_c$, the oscillators behave as if they are uncoupled. The phases become uniformly distributed and the coherence $r_t$ decays like $1/\sqrt{N}$.
- For all $\kappa > \widehat{\kappa}_c$, the incoherent state becomes unstable and $r_t$ grows to an eventual level $r_\infty < 1$. In the partially synchronized state, most oscillators co-rotate with the average phase $\psi_t$.
- As $\kappa \uparrow \infty$, synchronization increases and $r_\infty$ gets closer to 1.

A good review of these results can be found in the 2000 paper of S. H. Strogatz and also in,
The Kuramoto model : A simple paradigm for synchronization phenomena
by Acebrón, Bonilla, Pérez, Ritort, Spigler (Review of modern physics, 2005).

Given the phase $\theta^k$, the empirical measure $\mu_t^N$ is defined by,

$$\mu_t^N = \frac{1}{N}\sum_{j=1}^{N} \delta_{\{\theta_t^j\}} \quad \Rightarrow \quad \frac{1}{N}\sum_{j=1}^{N} \sin(\theta_t^j - \theta_t^k) = \int \sin(y - \theta_t^k)\, \mu_t^N(\mathrm{d}y).$$

If we add a common Brownian motion $B_t$, the equation becomes,

$$\mathrm{d}\theta_t^k = \omega^k\, \mathrm{d}t + \kappa \int \sin(y - \theta_t^k)\, \mu_t^N(\mathrm{d}y)\, \mathrm{d}t + \sigma \mathrm{d}B_t, \qquad k = 1, \ldots, N.$$

As $N$ tends to infinity, if all oscillators are identical, one expects $\mu_t^N$ to converge to the law of the representative oscillator $X_t$. And we expect $X_t$ to satisfy the following McKean-Vlasov equation,

$$\boxed{\mathrm{d}X_t = \omega\, \mathrm{d}t + \kappa \int \sin(y - X_t)\, \mu_t(\mathrm{d}y)\, \mathrm{d}t + \sigma \mathrm{d}B_t,}$$

where $\mu_t = Law(X_t)$, and $\omega$ is drawn from the common law of $\omega^k$'s.

Kuramoto was motivated by the phenomenon of collective synchronization as studied by Winfree. Biological examples include :

- Networks of pacemaker cells in the heart : Peskin (1975), Michaels, Matyas, Jalife (1987) ;
- Circadian pacemaker cells in the suprachiasmatic nucleus of the brain : Liu, Weaver, Strogatz, Reppert (1997) ;
- Metabolic synchrony in yeast cell suspensions : Aldridge, Pye (1976) ;
- Congregations of synchronously flashing fireflies : Buck (1988).

There are also many examples in physics and engineering including

- Arrays of lasers : Yu *et. al* (1995) ;
- Microwave oscillator ; York, Compton (1991) ;
- Josephson junctions in superconducting : Wiesenfeld, Colet, Strogatz (1998).

# Mean Field Approach

# Outline

- We treat the system of oscillators as a particle system.
- Instead of positing the dynamics of the particles, we let the individual particles determine endogenously their behaviors by minimizing a cost functional and hopefully, settling in a Nash equilibrium.
- Once the search for equilibrium is recast in this way, equilibria are given by solutions of nonlinear forward-backward systems.
- They are characterized by a backward Hamilton-Jacobi-Bellman (HJB) equation coupled to a forward Fokker-Planck-Kolmogorov (FPK) equation, and in the probabilistic approach, by forward-backward stochastic differential equations.

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*Jeux à champ moyen*, Lasry and Lions, 2006, 2007

*Large population stochastic dynamic games*, Huang, Malhamé, Caines, 2006.

- There are $N$ many oscillators.
- $\theta_t^k$ is the phase of the $k$-th oscillator for $k \in \{1, \dots, N\}$, and set $\boldsymbol{\theta}_t = (\theta_t^k)_{k=1,\dots,N}$.
- Each follow

$$\mathrm{d}\theta_t^k = \alpha_t^k \mathrm{d}t + \sigma \mathrm{d}B_t^k, \quad \text{(before it was } \mathrm{d}X_t = \kappa \int \sin(y - X_t)\mu_t(\mathrm{d}y)\mathrm{d}t + \sigma \mathrm{d}B_t\text{)},$$

  where $\omega^k$'s are set to zero, $B_t^k$ are independent Brownian motions and the adapted processes $\boldsymbol{\alpha} := (\alpha_t^k)_{t\geq 0}$ are the controls exerted by the individual oscillators.
- Controls are chosen in order to simultaneously minimize their costs given by

$$\boldsymbol{\alpha} \mapsto J^k(\boldsymbol{\alpha}) := \mathbb{E} \int_0^\infty e^{-\beta t} \left[\kappa \, L(\theta_t^k, \boldsymbol{\theta}_t) + \tfrac{1}{2}(\alpha_t^k)^2\right] \mathrm{d}t.$$

- We look for a Nash equilibrium.
- $L(\theta_t^k, \boldsymbol{\theta}_t)$ is the interaction cost that is specified in the next slight.
- Constant $\kappa \geq 0$ models the strength of the interactions between the oscillators.

Recall that Kuramoto equation is

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\theta_t^k = \omega^k + \kappa \int \sin(y - \theta_t^k)\,\mu_t^N(\mathrm{d}y)$$

We set $\omega^k = 0$. Then, the trigonometric identity $2\sin^2(x/2) = 1 - \cos(x)$, implies that,

$$\int \sin(y-x)\,\mu_t^N(\mathrm{d}y) = -\frac{\mathrm{d}}{\mathrm{d}x}\int [1-\cos(y-x)]\,\mu_t^N(\mathrm{d}y) = -2\frac{\mathrm{d}}{\mathrm{d}x}\int \sin^2((y-x)/2)\,\mu_t^N(\mathrm{d}y).$$

So formally the Kuramoto equation is the gradient flow of the 'energy'

$$L(\theta_t^k, \boldsymbol{\theta}_t) := 2\int \sin^2((\theta_t^k - y)/2)\;\mu_t^N(\mathrm{d}y).$$

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*Synchronization of coupled oscillators is a game*, by Yin, Mehta, Meyn, Shanbhag, IEEE (2011).

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Recall that

- $L(\theta^k_t, \boldsymbol{\theta}_t) := 2\int \sin^2((\theta^k_t - y)/2)\ \mu^N_t(\mathrm{d}y)$, and $\mathrm{d}\theta^k_t = \alpha^k_t \mathrm{d}t + \sigma \mathrm{d}B^k_t$.
- The problem is to minimize $\boldsymbol{\alpha} \mapsto J^k(\boldsymbol{\alpha}) := \mathbb{E}\int_0^\infty e^{-\beta t}\ [\kappa\ L(\theta^k_t, \boldsymbol{\theta}_t) + \frac{1}{2}(\alpha^k_t)^2]\ \mathrm{d}t$.

So as $N$ tends to infinity :

- $\theta^k_t$'s become statically more similar and the 'representative' oscillator $X_t$ follows $\mathrm{d}X_t = \alpha_t + \sigma \mathrm{d}B_t$.
- $\mu^N_t$ converges to the law of the representative oscillator $X_t$.
- Then, $L(\theta^k_t, \boldsymbol{\theta}_t)$ converges to $L(X_t, \mu_t) = 2\int \sin^2((X_t - y)/2)\ \mu_t(\mathrm{d}y)$, where $\mu_t = Law(X_t)$.
- The problem is to minimize $\boldsymbol{\alpha} \mapsto J(\boldsymbol{\alpha}) := \mathbb{E}\int_0^\infty e^{-\beta t}\ [\kappa\ L(X_t, \mu_t) + \frac{1}{2}(\alpha_t)^2]\ \mathrm{d}t$, where $dX_t = \alpha_t \mathrm{d}t + \sigma \mathrm{d}B_t$ and $\mu_t = Law(X_t)$.

As $\mu_t^N$ should converge to the law of the representative oscillator $X_t$, the mean field limit of the Nash equilibrium problem with initial distribution $\mu$ is summarized as follows,

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1. Start with a deterministic flow of probability measures $\boldsymbol{\mu} = (\mu_t)_{t\geq 0}$ with $\mu_0 = \mu$.
2. Find the optimal control $\boldsymbol{\alpha}^{*,\boldsymbol{\mu}} = (\alpha_t^{*,\boldsymbol{\mu}})_{t\geq 0}$ minimizing,

$$\boldsymbol{\alpha} = (\alpha_t)_{t\geq 0} \quad \mapsto \quad J(\boldsymbol{\alpha}\,;\ \boldsymbol{\mu}) := \mathbb{E}\int_0^\infty e^{-\beta t}\,[\kappa\; L(X_t^{\boldsymbol{\alpha}}, \mu_t) + \tfrac{1}{2}(\alpha_t)^2]\;\mathrm{d}t,$$

   where $\mathrm{d}X_t^{\boldsymbol{\alpha}} = \alpha_t \mathrm{d}t + \sigma \mathrm{d}B_t$, $Law(X_0) = \mu_0$, and $L(x,\mu) := 2\ \int_{-\pi}^{\pi}\ \sin^2((x-y)/2)\ \mu(\mathrm{d}y)$.
3. Find the fixed point $\mu_t = Law(X_t^{\boldsymbol{\alpha}^{*,\boldsymbol{\mu}}})$.

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- We say that a flow of probability measures $\boldsymbol{\mu} = (\mu_t)_{t \geq 0}$ is a solution of the KMFG, if it solves the fixed-point problem described above.
- We say that a probability measure $\mu$ is a stationary solution of the KMFG, if $\mu_t \equiv \mu$ is a solution.

The following is a simple but useful fact follows from the symmetry of the problem.

For a probability measure $\mu$ and $z \in \mathbb{R}$, translated measure $\mu(\cdot\,;\ z)$ is given by,

$$\mu(B\,;\ z) = \mu(\{y\ :\ y + z \in B\}).$$

If a probability measure $\mu$ is a stationary solution of the KMFG, then all translated measures $\mu(\cdot\,;\ z)$ are also solutions.

We first note that for a given probability measure $\mu$,

$$\begin{aligned} L(x,\mu) &= 2\int_{-\pi}^{\pi} \sin^2((x-y)/2)\ \mu(\mathrm{d}y) \\ &= \int_{-\pi}^{\pi} [1-\cos(x-y)]\ \mu(\mathrm{d}y) \\ &= 1-\int_{-\pi}^{\pi} [\cos(x)\cos(y)+\sin(x)\sin(y)]\ \mu(\mathrm{d}y) \\ &= 1-\mu(\cos)\cos(x)-\mu(\sin)\sin(x), \end{aligned}$$

where

$$\mu(\cos) = \int_{-\pi}^{\pi} \cos(y)\ \mu(\mathrm{d}y), \qquad \mu(\sin) = \int_{-\pi}^{\pi} \sin(y)\ \mu(\mathrm{d}y).$$

Let $U$ be the uniform measure on the circle. Then,

$$L(x, U) = 1 - U(\cos)\cos(x) - U(\sin)\sin(x) \equiv 1.$$

Then, the control problem corresponding to the stationary flow $U$ is

$$\text{minimize} \quad \boldsymbol{\alpha} = (\alpha_t)_{t\geq 0} \; \mapsto \; J(\boldsymbol{\alpha}\, ; \; U) := \mathbb{E} \int_0^\infty e^{-\beta t} \left[\kappa + \tfrac{1}{2}(\alpha_t)^2\right] \mathrm{d}t.$$

Cleary the optimal solution is $\alpha^* \equiv 0$, and the optimal state is $\mathrm{d}X_t^* = 0\,\mathrm{d}t + \sigma \mathrm{d}B_t$. Hence, $X_t^* = X_0^* + \sigma B_t$ and as $Law(X_0^*) = U$, we have $Law(X_t^*) = U$ as well. Hence,

The uniform measure $U$ is a stationary solution of the KMFG for every parameter.

The influence of the given flow of probability measures $\boldsymbol{\mu} = (\mu_t)_{t\geq 0}$ on the control problem is through $L(\cdot, \mu_t)$. The algebraic calculation before indicates only $t \in [0,\infty) \mapsto \kappa(\mu_t(\cos)\ ,\ \mu_t(\sin)) =: (a_t, b_t)$ is relevant. Hence, we look for a fixed point of the map

$$\boldsymbol{\mu} \ \mapsto \ (a_t, b_t) \ \mapsto \ X^{*,\boldsymbol{\mu}} \ \mapsto \ \mu^* \ \mapsto \ \kappa(\mathbb{E}[\cos(X_t^{*,\boldsymbol{\mu}})], \mathbb{E}[\sin(X_t^{*,\boldsymbol{\mu}})]),$$

where $X_t^*$ is the optimal process with running cost $\ell(x) = -\kappa(a_t \cos(x) + b_t \sin(x))$.

**Theorem**

*Probability flow $\boldsymbol{\mu} = (\mu_t)_{t\geq 0}$ is a solution of the Kuramoto mean field game if and only if*

$$\mu_t(\cos) = \mathbb{E}[\cos(X_t^{*,\boldsymbol{\mu}})], \quad \textit{and} \quad \mu_t(\sin) = \mathbb{E}[\sin(X_t^{*,\boldsymbol{\mu}})], \qquad \forall t \geq 0.$$

Critical interaction parameter is $\kappa_c := \beta\sigma^2 + \sigma^4/2$.

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**Theorem (Sub-critical interaction : incoherence)**

*For $\kappa < \kappa_c$, there exist a positive constant $\rho > 0$ depending on $\beta, \sigma, \kappa$ such that for any $\mu_0$ satisfying $d(\mu_0 - U) \leq \rho$, there exists a solution $\boldsymbol{\mu} = (\mu_t)_{t\geq 0}$ of the Kuramoto mean field game with interaction parameter $\kappa$ with $\mu_0 = \nu$ and $\mu_t$ converges in law to the uniform distribution as $t$ tends to infinity. Hence, the uniform measure is locally stable.*

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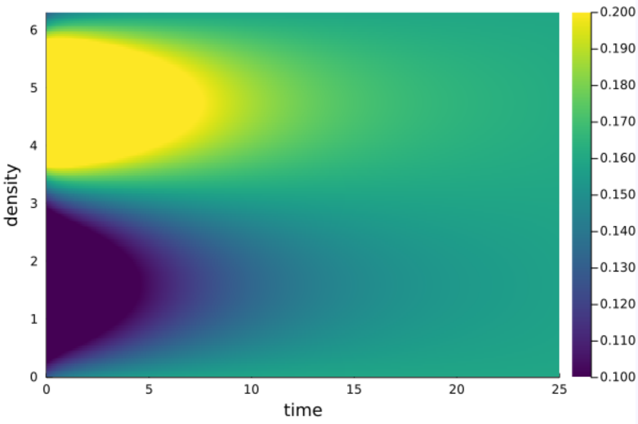
Proof constructs a fixed point of the map

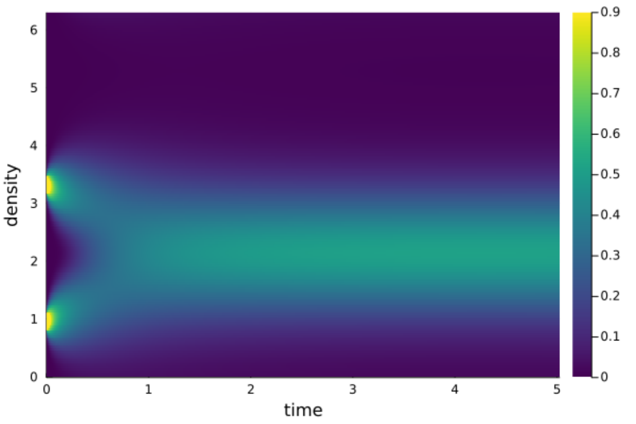
$$(a_t, b_t) \mapsto X^{*,\boldsymbol{\mu}} \mapsto \kappa(\mathbb{E}[\cos(X_t^{*,\boldsymbol{\mu}})], \mathbb{E}[\sin(X_t^{*,\boldsymbol{\mu}})]),$$

so that $(a_t, b_t)$ converges to zero exponentially, and the distance $d(\mu_0 - U)$ above is given by the action of $\mu_0 - U$ on five trigonometric functions.

We numerically compute the solutions of the Kuramoto mean field game with $\beta = \frac{1}{2}$, $\sigma = 1$ with critical value $\kappa_c = 1$. We consider $\kappa = 0.8 < \kappa_c$ with initial condition $\nu(\mathrm{d}x) = Ce^{-\sin(x)}\mathrm{d}x$. Below solution illustrates the convergence of the solution to the uniform distribution.



Again $\beta = \frac{1}{2}$, $\sigma = 1$ with critical value $\kappa_c = 1$. Now we consider $\kappa = 2 > \kappa_c$ with initial distribution that has two clusters around $\pi/2$ and $3\pi/2$. As seen below two clusters quickly merge and the solution converges towards a non-uniform invariant probability measure.



**Theorem (Super-critical interaction : synchronization)**

*For $\kappa > \kappa_c$, there exists a non-trivial stationary solutions of the KMFG.*

*Proof.* Suppose $\mu$ is a stationary solution. As all translations of $\mu$ are again a solution, by translating we may assume that $\mu(\sin) = 0$ and set $\gamma := \mu(\cos)$.
So we consider the control problem with the cost function $\kappa - \gamma \cos(x)$. Let $\mu_\gamma$ be the corresponding stationary measure of the optimal state process.
Set $F_\kappa(\gamma) := \int \cos(y)\ \mu_\gamma(\mathrm{d}y)$. Then, there is a solution if and only if $\kappa = F_\kappa(\gamma)$.
We compute that $F'(0) = \kappa/\kappa_c$. In particular, $F'(0) > 1$ when $\kappa > \kappa_c$.
As $F_\kappa(\gamma) \leq \kappa$, this implies the existence a fixed-point.

It is important that the critical value $\kappa_c$ is same in both proofs.

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**Lemma (Strong interaction : Full synchronization)**

*Let $\mu_n$ be a sequence of non-trivial stationary measures of the Kuramoto mean-field game with interaction parameters $\kappa_n$ tending to infinity. Then, there exists a sequence $z_n$ such that the translated stationary measures $\mu_n(\cdot\,;z_n)$ converge in law to the Dirac measure $\delta_{\{0\}}$.*

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*Proof.* We use viscosity solutions to study the dynamic programming equation :

$$\beta v^\gamma(x) - \frac{\sigma^2}{2} v^\gamma_{xx}(x) + \frac{1}{2}\,(v^\gamma_x(x))^2 = -\gamma\cos(x).$$

Suppose $\gamma \uparrow \infty$ and set $w^\gamma := \sqrt{\gamma}[v^\gamma + \gamma/\beta$. Then, $w^\gamma$ solves,

$$\beta_\gamma w^\gamma(z) - \frac{\sigma_\gamma^2}{2} w^\gamma_{xx}(z) + \frac{1}{2}(w^\gamma_x(z))^2 = 1 - \cos(z),$$

with $\beta_\gamma, \sigma_\gamma \to 0$. The limit Eikonal equation $w_x(z)^2/2 = 1 - \cos(z)$ has an explicit solution.

# An Application : Jet Lag Recovery

# Circadian rhythm

- Circadian rhythm is the oscillatory behavior of biological processes with a period close to 24 hours.
- Examples of circadian rhythms in animals include sleep/wake patterns, eating schedules, bodily temperatures, hormone production, and brain activity.
- These oscillations can be entrained to the 24 hour cycle of sunlight exposure.
- Abrupt disruptions, such as when an individual travels across time zones, results in jet lag.
- A region in the brain called Suprachiasmatic Nucleus (SCN) controls circadian rhythms.
- It contains on the order of $10,000$ neuronal oscillator cells,
- Each of these cells has a preferred frequency of slightly longer than 24 hours.

The following modification of the Kuramoto model is proposed for jet lag for SCN oscillators :

$$\frac{\mathrm{d}}{\mathrm{d}t}\theta_t^k = \omega^k + F\,\sin(\omega_s t + p(t) - \theta_t^k) + \frac{\kappa}{N}\sum_{j=1}^{N}\sin(\theta_t^j - \theta_t^k), \quad k = 1, \ldots, N,$$

- $\omega_s = 2\pi/24$ is the frequency of the external drive (which is sunlight),
- $F$ is the strength of the external drive,
- $p(t)$ is a phase shift accounting for the time zone angle at time $t$.
- $p(t) = p$ is used for an individual that stays in their time zone forever,
- whereas $p(t)$ increases for eastward travel and decreases for westward travel.

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*Resynchronization of circadian oscillators and the east-west asymmetry of jet-lag*,
by Lu, Cardena, Lee, Antonsen, Girvan, Ott, (2016).

- Suppose that one is a one time zone corresponding to a shift $p_1$ until time $\tau$ and moves to $p_2$.
- Then, the forcing function is given by,

$$p(t) = \begin{cases} p_1 & \text{if } t \leq \tau, \\ p_2 & \text{if } t > \tau. \end{cases}$$

- If $\kappa$, $F$, $\tau$ are large, the oscillators synchronize around $p_1$ with a period close to 24 hours. Then, at time $\tau$ abrupt disruption occurs and the oscillators gradually shift the phase to $p_2$.
- The paper studies the relaxation time of the transition and numerically finds larger recovery time for eastward travel.

- The interaction cost, has an extra term :

$$L(\theta_t^k, \boldsymbol{\theta}_t) = \ell(\theta_t^k, \mu_t^N) + c_{\text{sun}}(t, p(t), \theta_t^k),$$

- where the first term is interaction between oscillators

$$\ell(\theta_t^k, \mu_t^N) = 2\frac{1}{N}\sum_{j\neq i} \sin^2\left((\theta_t^k - \theta_t^j)/2\right) = 2\int_{-\pi}^{\pi} \sin^2\left((\theta_t^k - \theta)/2\right)\, \mu_t^N(\mathrm{d}\theta),$$

the empirical function $\mu_t^N$ is as before.

- the second term is the interaction with the external drive

$$c_{\text{sun}}(t, p(t), \theta_t^k) = 2\sin^2\left((\omega_s t + p(t) - \theta_t^k)/2\right),$$

function $p(t)$, $\omega_s$ as in the previous model.

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*Jet Lag Recovery : Synchronization of Circadian Oscillators as a Mean Field Game*, Carmona, Graves, (2020).

- Mean Field formalism have exactly the same solution structure as the dynamical system approach.
- As the uniform solutions are the desynchronized states, our results indicate a bifurcation from inhorence to self-organization at $\kappa_c$, and then convergence to full synchronization for very large interaction parameters.

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